ADIABATIC SELF-COMPRESSION OF GUN-PRODUCED PLASMA TORUS

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Abstract

Possibility of raising plasma temperature of gun-produced plasma torus by adiabatic self-compression is studied. Under certain operating conditions. the plasma temperature can be reached to more than four times of its initial value within time scale of one microsecond. It is proved that the equipartition between poloidal and toroidal magnetic energy is a consequence of plasma torus reaching its final equilibrium state without applied magnetic field. The plasma torus is expected MHD stable during the process of self-compression.

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I. Introduction

Research on magnetic confinement plasmas for fusion energy have now attained a certain stage: Tokamak might be developed to the first generation of fusion reactors using D+T reaction, simply because of its highest $n\tau T$ value already attained in laboratories as compared with other approaches. But satisfying the temperature, density and confinement time conditions for fusion is only part of the story. As considering with those facts such as ease of maintenance, overall efficiency, total plant cost, environment impact, etc., Tokamak seems not quite idealistic. Besides these, the engineering development of Tokamak reactor still has a long way to go. Therefore, researches on alternative concepts are still going on and worked quite actively during recent years.[1—4] They all seem to have one or more potential advantages over tokamaks despite of their relatively low experimental parameters attained now. New ideas continue to grow up. It is generally believed that some of them are worthwhile to develop as candidates of second or further generations of fusion reactors.

One of these alternate groups is called compact torus without applied toroidal field. Under certain operating conditions, they are MHD stable. The compact torus has a potential advantage to use advanced fuels, since they have much weak magnetic field and thus less cyclotron radiation loss as compared with tokamaks in reactor conditions.

Heating solely by ohmic current seems not enough to attain the ignition temperature, so various kinds of supplementary heating are indeed necessary to develop. Adiabatic compression is one of the inexpensive ways of heating the plasmas. In tokamaks, the convenient way for adiabatic compression is to raise the vertical field and push the plasma torus to much smaller major radius,[5—6] consequently the minor radius also contracts according to the conservation of magnetic flux. The potential of adiabatic compression in compact torus has not been considered before. In this paper it will be shown that under certain parameters, gun-produced plasma could be self-compressed to high enough temperature. Contrary to the tokamaks, the toroidal plasma here is not compressed by increasing the applied vertical field rather by the non-equilibrium electromagnetic force of the torus itself.

A gun-produced toroidal plasma can be produced by an annular discharge, which contributes to the toroidal field. It also captures a certain amount of poloidal magnetic energy near the muzzle of the discharge tube, which means induced toroidal current there. Generally speaking, the plasma is not in equilibrium along the major radius. Usually the toroidal magnetic energy is made much larger than the poloidal magnetic energy. The poloidal beta (β_p) is smaller than zero and the pressure outside the torus is larger than that inside. Since there is no applied vertical field there, the plasma leaves the muzzle of the gun and contracts along the major radius. We will show later that part of the toroidal magnetic energy transfers to the poloidal magnetic energy during the contraction period. Finally, it reaches an equilibrium position with nearly equipartition between these two magnetic energies. So equipartition of magnetic energies is a consequence of toroidal equilibrium along the major radius as well as along the minor radius.

II. Basic Formulation of Self-Compression Process

For the sake of exploring the physical meaning more clearly, we used the skin current model and large aspect ratio assumption. Since the self-compression process is fast enough, the ideal MHD formulations are justified here. The minor radius is much smaller than the major radius; thus, equilibrium along the minor radius is nearly satisfied and only the dynamic contraction process along the major radius needs to be considered.

The conservation of the toroidal flux within plasma torus is (MKS system)

$$\phi_t \propto B_t a^2 \propto \frac{I_p a^2}{R} = \text{const.} \tag{1}$$

so

$$\frac{dI_p}{I_p} + \frac{2da}{a} - \frac{dR}{R} = 0 \tag{1a}$$

here

I_p	total poloidal current
a	minor radius
R	major radius

The conservation of total poloidal flux outside the plasma torus is

$$\phi_p \propto R\left(\ln 8\frac{R}{a} - 2\right) I_t = \text{const.} \tag{2}$$

so

$$\frac{dI_t}{I_t} + \frac{dR}{R} + \frac{\frac{dR}{R} - \frac{da}{a}}{\ln 8\frac{R}{a} - 2} = 0 \tag{2a}$$

I_t	total toroidal current

for general polytropic compression

$$pV^{\alpha} \propto p\left(a^2R\right)^{\alpha} = \text{const.} \tag{3}$$

so

$$\frac{dp}{p} + \alpha\left(\frac{2da}{a} + \frac{dR}{R}\right) = 0. \tag{3a}$$

p pressure

A polytropic index equals $\frac{5}{3}$ for the adiabatic case and one for the isothermal case

$$\frac{5}{3} \leq \alpha \leq 1.$$

The force balance along the minor radius is

$$p - p_o = \frac{\mu I_t^2}{8\pi^2 a^2} - \frac{\mu I_p^2}{8\pi^2 R^2}. \tag{4}$$

Here p_o is the pressure outside the torus so

$$\frac{dp}{p} = \frac{\frac{\mu}{4\pi^2}\left[\frac{I_t^2}{a^2}\left(\frac{dI_t}{I_t} - \frac{da}{a}\right) - \frac{I_p^2}{R^2}\left(\frac{dI_p}{I_p} - \frac{dR}{R}\right)\right]}{p_o + \frac{\mu I_t^2}{8\pi^2 a^2} - \frac{\mu I_p^2}{8\pi^2 R^2}}. \tag{4a}$$

The equation of momentum along the major radius is

$$M\frac{dv}{dt} = M\frac{d(\frac{v^2}{2})}{dR} = \frac{\mu I_t^2}{2}\left(\ln 8\frac{R}{a} - \frac{3}{2} + \beta_p + \frac{\ell i}{2}\right). \tag{5}$$

Here

M	$\langle n \rangle m_i \times \pi a^2 \times 2\pi R$	
$\langle n \rangle$	average number density	
m_i	mass of ion	
β_p	poloidal beta	$(= 1 - \frac{a^2}{R^2}\frac{I_p^2}{I_t^2})$
l_i	internal inductance for skin current sheet model $l_i = 0$	

from (1a) and (4a), we obtain

$$
\begin{aligned}
\frac{dI_t}{dR} \times \frac{R}{I_t} = \Bigg\{ \Bigg[& (1 + \frac{1}{ln8R/a - 2}) \times 2a \times (p_0 + \frac{I_t^2 \mu}{8\pi^2 a^2} - \frac{\mu I_p^2}{8\pi^2 R^2}) \\
& + \left(\frac{1}{ln8R/a - 2} \times \frac{\mu I_p^2}{4\pi^2 R^2} \right) \\
& + \frac{1}{ln8R/a - 2} \times a \times \left(p_0 + \frac{\mu I_t^2}{8\pi^2 a^2} - \frac{\mu I_p^2}{8\pi^2 R^2} \right) \\
& - \left(1 + \frac{1}{ln8R/a - 2} \right) \frac{\mu I_t^2}{4\pi^2 a^2} \Bigg] - \frac{1}{ln8R/a - 2} + 2 \times \left(1 + \frac{1}{ln8R/a - 2} \right) \Bigg\} \div \\
& \Bigg\{ - \Bigg[2a \times \left(p_0 + \frac{\mu I_t^2}{8\pi^2 a^2} - \frac{\mu I_p^2}{8\pi^2 R^2} \right) + \frac{\mu I_t^2}{2\pi^2 R^2} - \frac{\mu I_t^2}{4\pi^2 a^2} \Bigg] \\
& + \frac{1}{ln8R/a - 2} \Bigg\}
\end{aligned} \tag{6}
$$

$$
\frac{dI_p}{dR} \times \frac{R}{I_p} = -2\left(ln8R/a - 2 \right) \times \left(\frac{R}{I_t} \frac{dI_t}{dR} \right) - \left[1 + 2(ln8R/a - 2) \right] \tag{7}
$$

$$
\frac{da}{dR} \times \frac{R}{a} = \frac{1}{2}\left(1 - \frac{dI_p}{dR} \times \frac{R}{I_p} \right) \tag{8}
$$

$$
\frac{dp}{dR} \times \frac{R}{p} = -a\left[1 + 2\left(\frac{da}{dR} \times \frac{R}{a} \right) \right] \tag{9}
$$

the whole compression process can be obtained by numerical integration Eqs. (6)-(9), and the final equilibrium state without vertical field is reached at

$$
ln8R/a - \frac{3}{2} + \beta_p = 0 \tag{10}
$$

$$
\beta_p = -ln8R/a + \frac{3}{2} < 0 \tag{11}
$$

since

$$
\beta_p = 1 - \left(\frac{a}{R} \frac{I_p}{I_t} \right)^2
$$

so

$$
\frac{I_p}{I_t} = \frac{R}{a}(1 - \beta_p)^{1/2} = \frac{R}{a}(ln8R/a - 0.5)^{1/2} \tag{12}
$$

the ratio between total toroidal and poloidal magnetic energy is

$$\frac{W_p}{W_t} = \frac{B_t^2/2\mu \times \pi a^2 \times 2\pi R}{1/2 L_t I_t^2}$$

here, $L_t = \mu R(ln8R/a - 2)$ so

$$\begin{aligned}
\frac{W_t}{W_p} &= \frac{1/2\mu \times (\mu I_p/2\pi R)^2 \times \pi a^2 \times 2\pi R}{1/2 \times \mu R \times (ln8R/a - 2) I_t^2} \\
&= \frac{I_p^2/I_t^2 \times a^2/R^2}{2 \times (ln8R/a - 2)} = \frac{ln8R/a - 0.5}{2(ln8R/a - 2)} \\
&\approx 1 \text{ (for wide range of} \frac{R}{a}) \qquad (13)
\end{aligned}$$

it is shown above that the equipartition of magnetic energy is a consequence of toroidal plasma equilibrium without applied vertical and toroidal field.

Now what we are interested is the possibility to raise the temperature high enough by self-compression. Since the parameters that could be reached in the final state rather than the details of the process are interested, we can ignore the integration of the whole set of differential equations and calculate the final state directly from the following set of algebraic equations,

conservation of magnetic fluxes

$$\psi_{ps} = \psi_{p1} \qquad (14a)$$

$$\psi_{t2} = \psi_{t1} \qquad (14b)$$

Equilibrium conditions at state 2

$$\begin{aligned}
\beta_{p2} &= 1 - \left(\frac{a}{R}\frac{I_{p2}}{I_{t2}}\right)^2 \\
&= -ln8\frac{R_2}{a_2} + \frac{3}{2} \qquad (14c)
\end{aligned}$$

conservation of energy

$$W_{p2} + W_{t2} + \int_1^2 pdV = W_{t1} + W_{p1} \qquad (14d)$$

or

$$I_{t1}R_1\left(ln8\frac{R_1}{a_1}-2\right)=I_{t2}R_2\left(ln8\frac{R_2}{a_2}-2\right) \tag{15a}$$

$$\frac{I_{p1}}{R_1}a_1^2=\frac{I_{p2}}{R_2}a_2^2 \tag{15b}$$

$$\frac{I_{p2}}{I_{t2}}=\frac{R_2}{a_2}\left(ln8\frac{R_2}{a_2}-0.5\right)^{1/2} \tag{15c}$$

$$\begin{aligned}&\frac{\left(\frac{\mu I_{p1}}{2\pi R_1}\right)^2}{2\mu}\times 2\pi R_1\times\pi a_1^2+\mu I_{t1}R_1\left(ln8\frac{R_1}{a_1}-2\right)\\ &=\frac{\left(\frac{\mu I_{p2}}{2\pi R_2}\right)^2}{2\mu}\times 2\pi R_2\times\pi a_2^2+\mu I_{t2}R_2\left(ln8\frac{R_2}{a_2}-2\right)\\ &+\frac{p_1(2\pi R_1\times\pi a_1^2)^k}{k-1}\left[\frac{1}{(2\pi R_2\times\pi a_2^2)^{k-1}}-\frac{1}{(2\pi R_1\times\pi a_1^2)^{k-1}}\right]\end{aligned} \tag{15d}$$

As an example, there is a gun-produced plasma, the initial state is

$$\begin{aligned}R_1&=1\,m\\ a_1&=0.1\,m\\ B_{t1}&=1\text{ Tesla}\\ B_{p1}&=0.1\text{ Tesla}\end{aligned}$$

where the subscript 1,2 means initial, final state. Clearly, the plasma torus is not equilibrium along the major radius in state 1. The total poloidal current

$$I_{p1}=\frac{B_{t1}\times 2\pi R}{\mu}=\frac{1\times 2\pi\times 1}{4\pi\times 10^{-7}}=5\times 10^6 A$$

the total toroidal current

$$\begin{aligned}I_{t1}&=\frac{B_{p1}\times 2\pi a_1}{\mu}\\ &=\frac{0.1\times 2\pi\times 0.1}{4\pi\times 10^{-7}}=5\times 10^4 A\end{aligned}$$

the total toroidal magnetic energy

$$\begin{aligned}W_{t1}&=\frac{B_{t1}^2}{2\mu}\times\pi a_1^2\times 2\pi R_1\\ &=\frac{(1)^2}{2\times 4\pi\times 10^{-7}}\times\pi\times(0.1)^2\times 2\pi\times 1=7.8\times 10^4 J\end{aligned}$$

the inductence

$$
\begin{aligned}
L_{t1} &= \mu R_1(ln8R_1/a_1 - 2) \\
&= 4\pi \times 10^{-7} \times 1 \times (ln8 \times 1/0.1 - 2) = 3 \times 10^{-6} H
\end{aligned}
$$

the total poloidal magnetic energy

$$
\begin{aligned}
W_{p1} &= \frac{1}{2} L_{t1} I_{t1}^2 \\
&= \frac{1}{2} \times 3 \times 10^{-6} \times (5 \times 10^4)^2 = 3.75 \times 10^3 J
\end{aligned}
$$

the total poloidal magnetic flux

$$
\psi_{p1} = L_{t1} I_{t1} = 3 \times 10^{-6} \times 5 \times 10^4 = 0.15 \text{ Weber}
$$

the total toroidal magnetic flux

$$
\psi_{t1} = B_{t1} \times \pi a_1^2 = 1 \times \pi \times (0.1)^2 = 3.14 \times 10^{-2} \text{ Weber}
$$

After iterative calculations, we obtained the parameters of the final state

$$
\begin{aligned}
W_{p2} &= \frac{1}{2} L_{t2} I_{t2}^2 \\
&= \frac{1}{2} (L_{t1} I_{t1}) I_{t2} = 3.5 \times 10^4 J
\end{aligned}
$$

then

$$
\begin{aligned}
I_{t2} &= \frac{3.5 \times 10^4}{1/2 \times 0.15} = 4.67 \times 10^{-5} A \\
\mu_0 R_2(ln8R_2/a_2 - 2) &= \frac{\psi_{p2}}{I_{t2}} = \frac{\psi_{p1}}{I_{t2}} \\
&= \frac{1.5 \times 10^{-1}}{4.67 \times 10^5} = 0.32 \times 10^{-6} H
\end{aligned}
$$

assumed $R/a = 3$ (it should be checked later)

$$
R_2 = \frac{0.32 \times 10^{-6}}{4\pi \times 10^{-7} \times [ln(8 \times 3) - 2]} = 0.216m
$$

since in the final equilibrium state the magnetic energy is equipartition as shown before

$$
W_{t2} \approx W_{p2} = 3.5 \times 10^4 J
$$

since

$$
\begin{aligned}
W_{t2} &= \frac{B_{t2}^2}{2\mu} \times \pi a_2^2 \times 2\pi R_2 \\
&= \frac{B_{t2}a_2^2}{2\mu} \times B_{t2} \times 2\pi^2 R_2 \\
&= \frac{B_{t1}a_1^2}{2\mu} \times B_{t2} \times 2\pi^2 R_2
\end{aligned}
$$

so

$$
\begin{aligned}
B_{t2} &= \frac{W_{t2}}{B_{t1}a_1^2/2\mu \times 2\pi^2 R_2} \\
&= \frac{3.5 \times 10^4}{[1 \times (0.1)^2/8\pi \times 10^{-7}] \times 2\pi^2 \times 0.216} = 2.06 w/m^2
\end{aligned}
$$

according to the conservation of the magnetic flux

$$
a_2 = \frac{a_1}{(B_{t2}/B_{t1})^{1/2}} = \frac{0.1}{(2.06)^{1/2}} = 0.07m
$$

$R_2/a_2 = \frac{0.216}{0.07} \approx 3$ as assumed before. The mechanical work done to the plasma during adiabatic compression is

$$
W_M = \int_{V_2}^{V_1} p dV = \frac{p_1 V_1^k}{k-1}\left(\frac{1}{V_2^{k-1}} - \frac{1}{V_1^{k-1}}\right)
$$

with

$$
\begin{aligned}
\langle n \rangle_1 &= 5 \times 10^{19} m^{-3} \\
T_1 &= 10^{7\circ} k \\
p_1 &= 2\langle n \rangle_1 k T_1 = 1.38 \times 10^4 \frac{\text{Newton}}{m^2} \\
V_1 &= \pi(a_1)^2 \times 2\pi R_1 = 0.2 m^3 \\
V_2 &= \pi(a_2)^2 \times 2\pi R_2 = 0.0209 m^3 \\
k &= \frac{5}{3}
\end{aligned}
$$

$$
\begin{aligned}
&W_M = 1.45 \times 10^4 J \\
&W_{t1} + W_{p1} \approx W_{t2} + W_{p2} + W_M
\end{aligned}
$$

the final equilibrium β_{p2} without applied vertical field

$$
\begin{aligned}
\beta_{p2} &= -ln 8R_2/a_2 + \frac{3}{2} \\
&= -ln 24 + \frac{3}{2} \approx -1.7
\end{aligned}
$$

the temperature ratio

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = \left(\frac{a_1}{a_2}\right)^{2(k-1)}\left(\frac{R_1}{R_2}\right)^{k-1}$$
$$= \left(\frac{0.1}{0.07}\right)^{4/3}\left(\frac{1}{0.216}\right)^{2/3} = 4.5.$$

The time scale for self-compression can be estimated as follows, since

$$\beta_{p1} = 1 - \frac{a^2}{R^2}\frac{I_p^2}{T_t^2}$$
$$= 1 - \left(\frac{0.1}{1.0}\right)^2\left(\frac{5\times10^6}{5\times10^4}\right)^2 \approx -10^2$$

from

$$M\frac{dv}{dt} = \frac{\mu I_t^2}{2}\left(ln8R/a + \beta_p - \frac{3}{2} + \frac{li}{2}\right)$$
$$M\frac{\Delta R}{(\Delta t)^2} \approx \frac{\mu I_t^2}{2} \times \beta_p$$
$$\Delta t \approx \left[\frac{2M\Delta R}{\mu I_t^2 \beta_p}\right]^{1/2}$$
$$\approx 10^{-6}\ \text{sec.}$$

that means the toroidal plasma self-compressed from initial to final equilibrium state within order of one microsecond, as compared with the plasma loss within this time scale, the process could be really treated as an adiabatic compression process.

III. Stability Properties of the Plasma Torus[7]

Since self-compression could be treated as an evolutionary process of different near minor radius equilibrium configurations, it is necessary to check if it is MHD stable. Otherwise, the plasma will be destroyed and canot be compressed effectively. From the basic MHD stability theory of surface current model of large aspect ratio torus, if there is no applied toroidal field, the stability properties are determined by the ratio of induced poloidal and toroidal field $\frac{B_p}{B_t}$, if $\frac{B_z}{B_t} < \sqrt{2}$, it is stable to $m = 0$ mode; if $\frac{B_p}{B_t} < \frac{1}{ln R/a}$, it is stable to $m \geq 1$ for the largest possible wavelength. Obviously all these stable criteria can be satisfied from the initial to the final state of the self-compressed torus. Since $\frac{dp}{dr} > 0$ for this kind of torus, localized mode is also stabilized according to Mercier's criteria.

It should be noted that the poloidal magnetic field of gun-produced plasma should be reconnected in time after leaving the muzzle. Otherwise, the configuration could be quite elongated and quasi-cylindrical in nature, since the equilibrium and stability properties of quasi-cylindrical configurations are quite different from the quasi-circular configuration, the evolutionary process could be a different story.[8]

IV. Conclusions

1. Gun-produced plasma torus with different total toroidal and poloidal magnetic energy after leaving the muzzle of the gun is non-equilibrium along major radius. Under certain operating conditions it will contract along major radius as well as minor radius. The adiabatic self-compression process without a guiding field can raise the plasma temperature to 4 to 5 times its initial value within time scale of one microsecond.
2. Magnetic energy equipartition is a consequence of equilibrium of plasma torus without applied toroidal and vertical field.
3. The plasma torus is MHD stable during the whole self-compression period provided the reconnection will occur in time after the plasma leaves the muzzle of the gun.

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